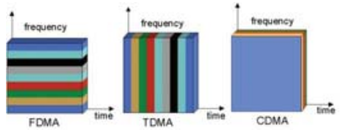

ECS455: Chapter 4

Multiple Access

4.6 Orthogonality and CDMA

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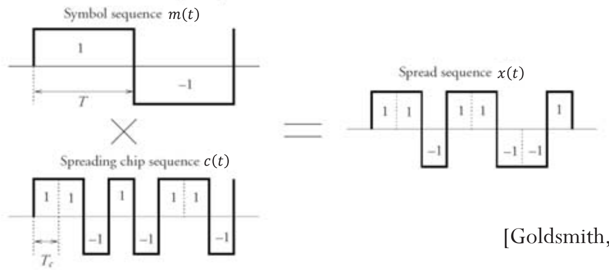
DSSS and m-sequences

- m-sequences
  - **Excellent auto-correlation** properties (for ISI rejection)
  - Highly **suboptimal** for exploiting the **multiuser** capabilities of spread spectrum.
- There are only a **small number** of maximal length codes of a given length.
- Moreover, maximal length codes generally have relatively **poor cross-correlation** properties, at least for some sets of codes.



[Goldsmith, 2005, Ch 13]

Number of primitive polynomials

Number of different primitive polynomials:

- ***r*** is the **degree** of the primitive polynomials and
- N_p is the number of different primitive polynomials available.

r	N_p	r	N_p
2	1	11	176
3	2	12	144
4	2	13	630
5	6	14	756
6	6	15	1800
7	18	16	2048
8	16	17	7710
9	48	18	8064
10	60	19	27594

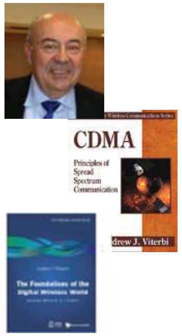
[Chen, 2007, p 145]

SSMA

- For spread spectrum systems with **multiple users**, codes such as Gold, Kasami, or Walsh codes are used instead of maximal length codes
- Superior cross-correlation properties.
- Worse auto-correlation than maximal-length codes.
  - The autocorrelation function of the spreading code determines its multipath rejection properties.

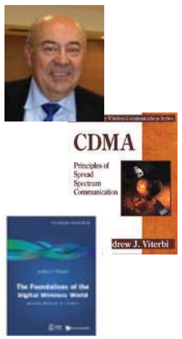
Qualcomm

- Founders: Two of the most eminent engineers in the world of mobile radio
- Prof. Irwin **Jacobs** is the chairman and founder
  - Cornell (undergrad.: Hotel > EE)
  - MIT (grad.)
  - UCSD (Prof.)
- Prof. Andrew J. **Viterbi** is the co-founder
  - MIT (BS, MS)
  - USC (PhD)
  - UCLA and UCSD (Prof.)
  - Same person that invented the Viterbi algorithm for decoding convolutionally encoded data.



Code Division Multiple Access (CDMA)

- **1991**: Qualcomm announced
  - that it had invented a new cellular system based on **CDMA**
  - that the capacity of this system was **20 or so times greater** than any other cellular system in existence
- However, not all of the world was particularly pleased by this apparent breakthrough—in particular, **GSM** manufacturers became concerned that they would start to lose market share to this new system.
  - The result was continual and vociferous argument between Qualcomm and the GSM manufacturers.



[Webb, 1998, p 138]

CDMA

- One way to achieve SSMA
- May utilize Direct Sequence Spread Spectrum (DS/SS)
  - The narrowband message signal is multiplied (modulated) by the **spreading signal** which has a very large bandwidth (orders of magnitudes greater than the data rate of the message).
  - Direct sequence is not the only spread-spectrum signaling format suitable for CDMA
- All users use the same carrier frequency and may transmit simultaneously.
- Users are assigned different "**signature waveforms**" or "code" or "codeword" or "**spreading signal**"
- Each user's codeword is ***approximately orthogonal*** to all other codewords.
- Should not be confused with the mobile phone standards called cdmaOne (Qualcomm's IS-95) and CDMA2000 (Qualcomm's IS-2000) (which are often referred to as simply "CDMA")
  - These standards use CDMA as an underlying channel access method.

Not to be confused with error-correcting codes that add redundancy to combat channel noise and distortion

Inner Product (Cross Correlation)

- Vector

$$\langle \vec{x}, \vec{y} \rangle = \vec{x} \cdot \vec{y}^* = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}^* = \sum_{k=1}^{n} x_k y_k^*$$

Complex conjugate

- Waveform: Time-Domain

$$\langle x, y \rangle = \int_{-\infty}^{\infty} x(t) y^*(t) dt$$

- Waveform: Frequency Domain

$$\langle X, Y \rangle = \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

Conjugation is not required when dealing only with real-valued signals.

Review: Orthogonality

- Two signals are said to be **orthogonal** if their **inner product** is zero.
- The symbol $\perp$ is used to denote orthogonality.

Vector:

$$\langle \vec{a},\vec{b}\rangle = \vec{a}\cdot\vec{b}^* = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}^* = \sum_{k=1}^{n} a_k b_k^* = 0$$

Time-domain:

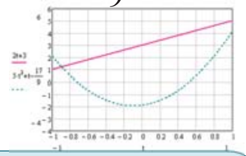
$$\langle a,b\rangle = \int_{-\infty}^{\infty} a(t)\, b^*(t)\,dt = 0$$

Frequency domain:

$$\langle A,B\rangle = \int_{-\infty}^{\infty} A(f)\, B^*(f)\,df = 0$$

Example:

$2t+3$ and $5t^2+t-\frac{17}{9}$ on $[-1,1]$



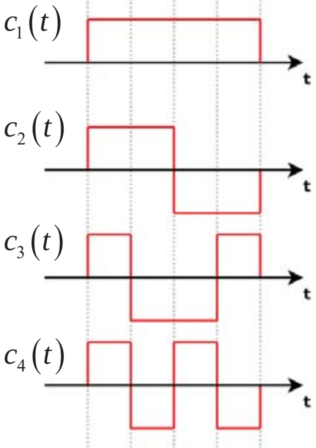
Example (Fourier Series):

$\sin\left(2\pi k_1 \frac{t}{T}\right)$ and $\cos\left(2\pi k_2 \frac{t}{T}\right)$ on $[0,T]$

$e^{j2\pi n\frac{t}{T}}$ on $[0,T]$

Ex: Orthogonal Signals

Waveform (cont.-time) version



When $i \neq j$, $\langle c_i(t), c_j(t)\rangle = 0$.

Vector (discrete-time) version

$$\underline{\mathbf{c}}^{(1)} = [+1 \quad +1 \quad +1 \quad +1]$$
$$\underline{\mathbf{c}}^{(2)} = [+1 \quad +1 \quad -1 \quad -1]$$
$$\underline{\mathbf{c}}^{(3)} = [+1 \quad -1 \quad -1 \quad +1]$$
$$\underline{\mathbf{c}}^{(4)} = [+1 \quad -1 \quad +1 \quad -1]$$

When $i \neq j$, $\langle \underline{\mathbf{c}}^{(i)}, \underline{\mathbf{c}}^{(j)}\rangle = 0$.

Review: Important Properties

- Parseval's theorem

$$\langle x,y\rangle \equiv \int_{-\infty}^{\infty} x(t)\,y^*(t)\,dt = \int_{-\infty}^{\infty} X(f)\,Y^*(f)\,df \equiv \langle X,Y\rangle$$

1. $x(t) \perp y(t)$ iff $X(f) \perp Y(f)$.
2. $E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$.

It is therefore sufficient to check only on the "convenient" domain.

- Useful observation: If the non-zero regions of two signals
  - do not overlap in time domain or (TDMA)
  - do not overlap in frequency domain, (FDMA)

  then the two signals are orthogonal (their inner product = 0).
- However, in general, orthogonal signals may overlap both in time and in frequency domain.

Orthogonality-Based MA

- Consider a system with ℓ users.
- Suppose that the kth user want to transmit a number S_k.
  - Could be a sample from his/her analog message.
  - Could be -1 or 1, representing message bit 1 or 0.
- We create multiple communication channels (with no inter-channel interference); one for each user.

CDMA

$$x(t) = \sum_{k=0}^{\ell-1} s_k c_k(t) \xrightarrow{\mathcal{F}} X(f) = \sum_{k=0}^{\ell-1} s_k C_k(f)$$

(***Orthogonal*** signaling) where $c_{k_1} \perp c_{k_2}$

- The k^{th} code (signal/waveform) is assigned to (used by) the k^{th} user.

Orthogonality-Based MA

CDMA

$$x(t)=\sum_{k=0}^{\ell-1} s_k c_k(t) \xrightarrow{\mathcal{F}} X(f)=\sum_{k=0}^{\ell-1} s_k C_k(f)$$

(***Orthogonal*** signaling)

where $c_{k_1} \perp c_{k_2}$

TDMA

$$x(t)=\sum_{k=0}^{\ell-1} s_k c(t-kT_s) \xrightarrow{\mathcal{F}} X(f)=C(f)\sum_{k=0}^{\ell-1} s_k e^{-j2\pi fkT_s}$$

where $c(t)$ is time-limited to $[0,T]$.

This is a special case of CDMA with $c_k(t)=c(t-kT_s)$

The c_k are non-overlapping in time domain.

FDMA

$$X(f)=\sum_{k=0}^{\ell-1} s_k C(f-k\Delta f)$$

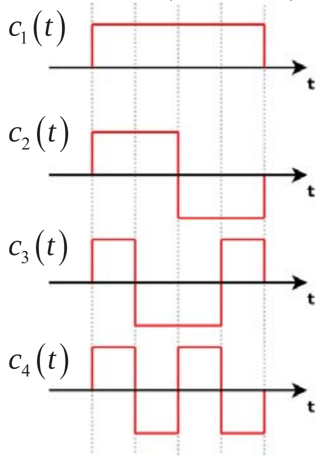
where $C(f)$ is frequency-limited to $[0,\Delta f]$.

This is a special case of CDMA with $C_k(f)=C(f-k\Delta f)$

The C_k are non-overlapping in freq. domain.

Ex: DS-CDMA

Waveform (cont.-time) version



$$x(t)=\sum_{k=1}^{4} s_k c_k(t)$$

Vector (discrete-time) version

$$\underline{\mathbf{c}}^{(1)}=[+1 \quad +1 \quad +1 \quad +1]$$
$$\underline{\mathbf{c}}^{(2)}=[+1 \quad +1 \quad -1 \quad -1]$$
$$\underline{\mathbf{c}}^{(3)}=[+1 \quad -1 \quad -1 \quad +1]$$
$$\underline{\mathbf{c}}^{(4)}=[+1 \quad -1 \quad +1 \quad -1]$$

$$\left\langle \underline{\mathbf{c}}^{(i)},\underline{\mathbf{c}}^{(j)}\right\rangle=\begin{cases}4, & i=j,\\ 0, & i\neq j.\end{cases}$$

$$\underline{\mathbf{x}}=\sum_{k=1}^{4} s_k \underline{\mathbf{c}}^{(k)}$$

Ex: DS-CDMA (Uplink)

$$\underline{\mathbf{c}}^{(1)}=[+1 \quad +1 \quad +1 \quad +1]$$
$$\underline{\mathbf{c}}^{(2)}=[+1 \quad +1 \quad -1 \quad -1]$$
$$\underline{\mathbf{c}}^{(3)}=[+1 \quad -1 \quad -1 \quad +1]$$
$$\underline{\mathbf{c}}^{(4)}=[+1 \quad -1 \quad +1 \quad -1]$$

- At the transmitter (mobile phone) of each user:
  - User 1's message is $s_1 = 14$.
    - Transmit $s_1 \times \underline{\mathbf{c}}^{(1)}$
  - User 2's message is $s_2 = 20$.
    - Transmit $s_2 \times \underline{\mathbf{c}}^{(2)}$
  - User 3's message is $s_3 = 26$.
    - Transmit $s_3 \times \underline{\mathbf{c}}^{(3)} = 26 \times [+1 \quad -1 \quad -1 \quad +1] = [26 \quad -26 \quad -26 \quad 26]$
  - User 4's message is $s_4 = -5$.
    - Transmit $s_4 \times \underline{\mathbf{c}}^{(4)} = -5 \times [+1 \quad -1 \quad +1 \quad -1] = [-5 \quad 5 \quad -5 \quad 5]$
- In the air,
  - the signals from all the users are combined to create
    - $\underline{\mathbf{x}} = \sum_{k=0}^{\ell-1} s_k \underline{\mathbf{c}}^{(k)} = [55 \quad 13 \quad -37 \quad 25]$
  - Additionally, the signal may be further corrupted by the noise and fading.
    - $\underline{\mathbf{r}} = \mathbf{H}\underline{\mathbf{x}} + \underline{\mathbf{n}}$
    - However, here, we will ignore such corruption for clearer MA calculation.

Ex: DS-CDMA (Uplink)

- At the receiver (base station),
  - $\underline{\mathbf{r}} = \underline{\mathbf{x}} = \sum_{k=1}^{4} s_k \underline{\mathbf{c}}^{(k)} = [s_1 \quad s_2 \quad s_3 \quad s_4]\begin{bmatrix}\underline{\mathbf{c}}^{(1)}\\ \underline{\mathbf{c}}^{(2)}\\ \underline{\mathbf{c}}^{(3)}\\ \underline{\mathbf{c}}^{(4)}\end{bmatrix} = \underline{\mathbf{s}}\mathbf{C}$
  - To find s_3, note that
    - $\langle \underline{\mathbf{r}}, \underline{\mathbf{c}}^{(3)}\rangle = \underline{\mathbf{r}} \cdot \underline{\mathbf{c}}^{(3)} = \left(\sum_{k=1}^{4} s_k \underline{\mathbf{c}}^{(k)}\right) \cdot \underline{\mathbf{c}}^{(3)} = \sum_{k=1}^{4} s_k \left(\underline{\mathbf{c}}^{(k)} \cdot \underline{\mathbf{c}}^{(3)}\right)$

$$\langle \underline{\mathbf{c}}^{(i)}, \underline{\mathbf{c}}^{(j)}\rangle = \begin{cases}4, & i=j,\\ 0, & i\neq j.\end{cases}$$

  - In general, for orthogonal codes containing only ± 1,
  - Observe to recover s_k, we only need $\underline{\mathbf{c}}^{(k)}$; we don't need to know the codes for other users.

Ex: DS-CDMA (Uplink)

- One can define $\hat{\underline{\mathbf{s}}} = [\hat{s}_1 \quad \hat{s}_2 \quad \hat{s}_3 \quad \hat{s}_4]$.
- Then,

$$\hat{\underline{\mathbf{s}}} = [\hat{s}_1 \quad \hat{s}_2 \quad \hat{s}_3 \quad \hat{s}_4] = \left[\frac{1}{N}\underline{\mathbf{r}}\cdot\underline{\mathbf{c}}^{(1)} \quad \frac{1}{N}\underline{\mathbf{r}}\cdot\underline{\mathbf{c}}^{(2)} \quad \frac{1}{N}\underline{\mathbf{r}}\cdot\underline{\mathbf{c}}^{(3)} \quad \frac{1}{N}\underline{\mathbf{r}}\cdot\underline{\mathbf{c}}^{(4)}\right]$$

$$= \left[\frac{1}{N}\underline{\mathbf{r}}\left(\underline{\mathbf{c}}^{(1)}\right)^T \quad \frac{1}{N}\underline{\mathbf{r}}\left(\underline{\mathbf{c}}^{(2)}\right)^T \quad \frac{1}{N}\underline{\mathbf{r}}\left(\underline{\mathbf{c}}^{(3)}\right)^T \quad \frac{1}{N}\underline{\mathbf{r}}\left(\underline{\mathbf{c}}^{(4)}\right)^T\right]$$

$$= \frac{1}{N}\underline{\mathbf{r}}\left[\left(\underline{\mathbf{c}}^{(1)}\right)^T \quad \left(\underline{\mathbf{c}}^{(2)}\right)^T \quad \left(\underline{\mathbf{c}}^{(3)}\right)^T \quad \left(\underline{\mathbf{c}}^{(4)}\right)^T\right] = \frac{1}{N}\underline{\mathbf{r}}\begin{bmatrix}\underline{\mathbf{c}}^{(1)} \\ \underline{\mathbf{c}}^{(2)} \\ \underline{\mathbf{c}}^{(3)} \\ \underline{\mathbf{c}}^{(4)}\end{bmatrix}^T = \frac{1}{N}\underline{\mathbf{r}}\mathbf{C}^T$$

CDMA's key equation: $\underline{\mathbf{s}} = \frac{1}{N}(\underline{\mathbf{s}}\mathbf{C})\mathbf{C}^T$

Key property of C

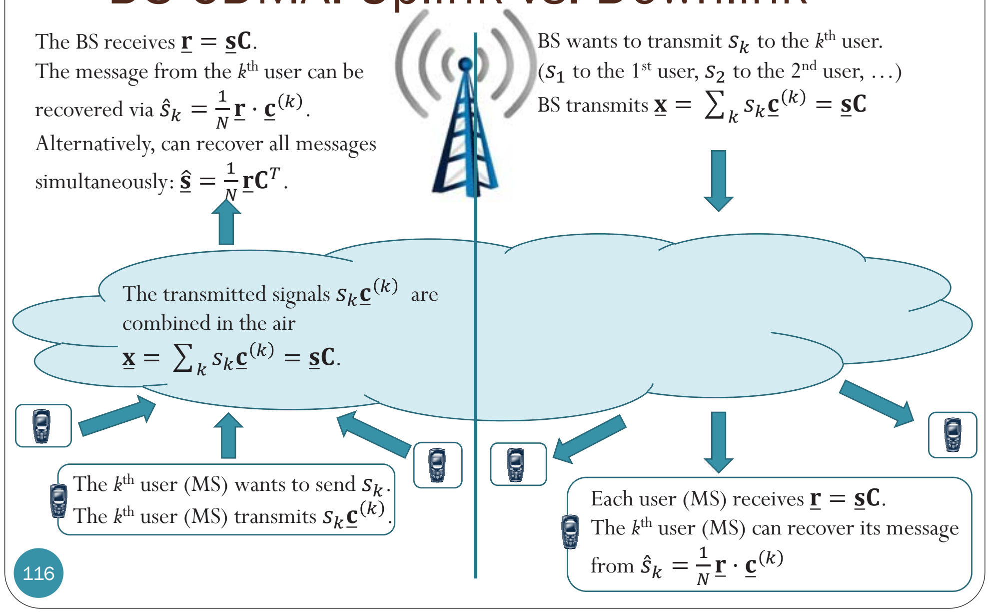
- From the CDMA's key equation $\underline{\mathbf{s}} = \frac{1}{N}(\underline{\mathbf{s}}\mathbf{C})\mathbf{C}^T$, or from the fact that all the rows of $\mathbf{C}$ are orthogonal,
- we have the key property of $\mathbf{C}$:

$$\mathbf{C}\mathbf{C}^T = N\mathbf{I}.$$

- It is tempting to call this an orthogonal matrix.
  - However, in linear algebra, to have an orthogonal matrix, the matrix must satisfy
    1. the rows are orthogonal and
    2. the rows must be unit vectors.

    In other words, the rows must be orthonormal vectors. Equivalently, The matrix must satisfy $\mathbf{A}\mathbf{A}^T = \mathbf{A}^T\mathbf{A} = \mathbf{I}$.

DS-CDMA: Uplink vs. Downlink



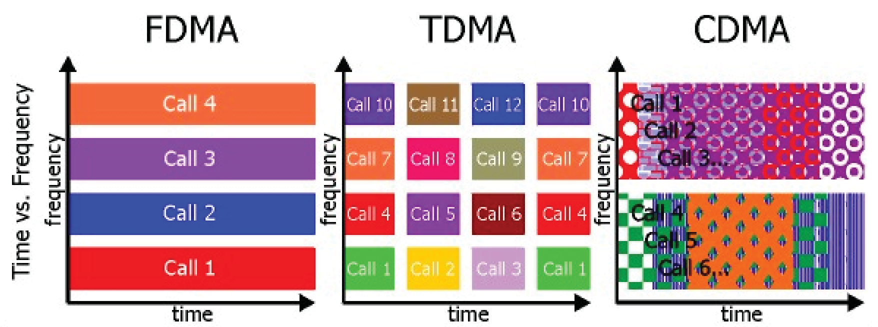
CDMA: DS/SS

- The receiver performs **a time correlation operation** to detect only the specific desired codeword.
- All other codewords appear as noise due to decorrelation.
- For detection of the message signal, the receiver needs to know the codeword used by the transmitter.
- **Each user operates independently with no knowledge of the other users.**
- Unlike TDMA or FDMA, CDMA has a **soft capacity limit**.
  - Increasing the number of users in a CDMA system raises the noise floor in a linear manner.
  - There is no absolute limit on the number of users in CDMA. Rather, the system performance gradually degrades for all users as the number of users is increased and improves as the number of users is decreased.

[Rappaport, 2002, Ch9]

Analogy [Tanenbaum, 2003]

- An airport lounge with many pairs of people conversing.
- TDMA is comparable to all the people being in the middle of the room but taking turns speaking.
- FDMA is comparable to the people being in widely separated clumps, each clump holding its own conversation at the same time as, but still independent of, the others.
- CDMA is comparable to everybody being in the middle of the room talking at once, but with each pair in a different language.
  - The French-speaking couple just hones in on the French, rejecting everything that is not French as noise.
  - Thus, the key to CDMA is to be able to extract the desired signal while rejecting everything else as random noise.



Conversation Analogy	FDMA	TDMA	CDMA
	Everyone talks in a different room to prevent interference. Since the conversation can't be heard from another room, it can be filtered from the other by going to the other room.	Within each room, everyone takes turns talking to prevent interference. Within each room, one person is talking at once, so they must talk fast to say everything.	Everyone speaks a different language at the same time in the same room. Since each language is unique, one may be filtered from another.

CDMA: Near-Far Problem

- At first, CDMA did **not** appear to be **suitable** for mobile communication systems because of this problem.
- Occur when many mobile users share the same channel.
- In an **uplink**, the signals received from each user at the receiver travel through different channels.
- **Users that are close to the BS can cause a great deal of interference to user's farther away**.
  - In general, the strongest received mobile signal will **capture** the demodulator at a base station.
- Stronger received signal levels raise the noise floor at the base station demodulators for the weaker signals, thereby decreasing the probability that weaker signals will be received.
- Fast **power control** mechanisms solve this problem.
  - Regulate the transmit power of individual terminals in a manner that received power levels are **balanced** at the BS.

How many orthogonal signals?

- No signal can be both strictly time-limited and strictly band-limited.
- We adopt a softer definition of bandwidth and/or duration (e.g., the percentage of energy outside the band [-B, B] or outside the time interval [0,T] not exceeding a given bound ε.
- Q: How many mutually orthogonal signals with (approximate) duration T and (approximate) bandwidth B can be constructed?
- A: About 2TB
  - No explicit answer in terms of T, B, and ε is known.
  - Unless the product TB is small.
- A K-user orthogonal CDMA system employing antipodal modulation at the rate of R bits per second requires bandwidth approximately equal to

$$B = \frac{1}{2} RK$$

[Verdu, 1998, Ch1, p 7]